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sets of values are $\begin{cases} c = 1 \\ a = 1 \end{cases}$, $\begin{cases} c = 2 \\ a = 1 \end{cases}$, $\begin{cases} c = 3 \\ a = 2 \text{ or } 4 \end{cases}$, etc. The first set is the one given in the problem. The second gives $m'/n' = (m + 4n)/(2m + n)$ which yields the series $1/1, 5/3, 17/13, 69/47, \dots$

(b) Almost the same work leads to similar results in the case of any surd, \sqrt{k} . We find $a = d$, $b = kc$, $2a \geq c(k - 1)$, $kc^2 > a^2$. These possess, among others, the following solutions for the case where $k = 3$.

$$\begin{cases} c = 1 \\ a = 1 \end{cases}, \begin{cases} c = 2 \\ a = 3 \end{cases}, \begin{cases} c = 3 \\ a = 4 \text{ or } 5 \end{cases}, \text{ etc.}$$

The first set gives $m'/n' = (m + 3n)/(m + n)$, which yields the series $1/1, 4/2 = 2/1, 5/3, 14/8 = 7/4, 19/11, \dots$

Also solved by NORMAN ANNING.

GEOMETRY.

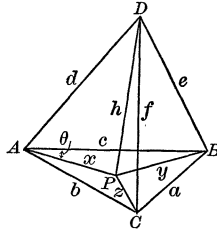
466. Proposed by HORACE OLSON, Chicago, Illinois.

Given the edges of a triangular pyramid, find the radius of the inscribed sphere.

SOLUTION BY A. H. HOLMES, Brunswick, Maine.

Calling the radius of the sphere r , and using the notation in the figure, we have,

$$h^2 = d^2 - x^2 = c^2 - y^2 = f^2 - z^2, \\ \cos \theta = \frac{c^2 + x^2 - y^2}{2cx} = \frac{c^2 + d^2 - e^2}{2cx} = \frac{n}{x}. \quad (x \cos \theta = n.)$$



$$\cos(A - \theta) = \cos A \cos \theta + \sin A \sin \theta = \frac{b^2 + x^2 - z^2}{2bx} = \frac{b^2 + d^2 - f^2}{2bx} = \frac{m}{x}.$$

Hence,

$$\sin \theta = \frac{m - n \cos A}{x \sin A}, \quad x \sin \theta = \frac{m - n \cos A}{\sin A}.$$

Hence,

$$x^2 = n^2 + \frac{(m - n \cos A)^2}{\sin^2 A}, \quad \text{and} \quad h = \frac{\sqrt{(d^2 - n^2) \sin^2 A - (m - n \cos A)^2}}{\sin A}.$$

Hence,

$$3 \text{ times contents of pyramid} = \frac{bc}{2} \sqrt{(d^2 - n^2) \sin^2 A - (m - n \cos A)^2},$$

and therefore,

$$r = \frac{bc \sqrt{(d^2 - n^2) \sin^2 A - (m - n \cos A)^2}}{2(ABC + ACD + ABD + BCD)},$$

in which

$$m = \frac{b^2 + d^2 - f^2}{2b}, \quad n = \frac{c^2 + d^2 - e^2}{2c}, \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc},$$

$$\sin A = \sqrt{1 - \frac{(b^2 + c^2 - a^2)^2}{4b^2c^2}}, \quad ABC = \sqrt{s(s-a)(s-b)(s-c)},$$

and similarly for ABD , ACD and BCD .

Also solved by WALTER C. EELLS and J. W. CLAWSON.

CALCULUS.

380. Proposed by C. N. SCHMALL, New York City.

Show that

$$\int_0^\infty \left[\frac{1}{1^4 + x^2} + \frac{1}{2^4 + x^2} + \frac{1}{3^4 + x^2} + \cdots \right] dx = \frac{\pi^3}{12},$$

where the series in the brackets is infinite.

SOLUTION BY A. M. HARDING, University of Arkansas.

For all values of x in the interval $(0, \infty)$ the n th term of the given series has the property

$$\frac{1}{n^4 + x^2} \leq \frac{1}{n^4}.$$

Now the series $\sum_{n=1}^\infty 1/n^4$ converges. Hence, the given series is uniformly convergent in the interval $(0, \infty)$.

Each term of the series is continuous in the interval $(0, \infty)$.

Hence, it may be integrated term by term. Hence,

$$\int_0^\infty \left[\frac{1}{1^4 + x^2} + \frac{1}{2^4 + x^2} + \frac{1}{3^4 + x^2} + \cdots \right] dx = \frac{\pi}{2} \left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \right].$$

It can be easily shown that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}.$$

Hence, the given integral $= \frac{\pi^3}{12}$.

Also solved by S. A. JOFFE, and J. A. CAPARO.

299. Proposed by B. F. FINKEL, Drury College.

A cone rests in two fluids which do not mix, with its vertex downwards and its base in the surface of the upper fluid; to find how much its density must be increased that it may rest with its base in the common surface of the fluids. (From Walton's *Hydrostatical Problems*.)

SOLUTION BY J. F. BRACHO, University of Notre Dame.

Let, w = density of cone in first position, w_1 = density of cone in second position, d_1 = density of upper fluid, d_2 = density of lower fluid. We have:

$$DC = \frac{AB + OD}{OA} = \frac{r(h - a)}{h}.$$